A study of interpolation methods for forest fire danger rating in Canada

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Canadian fire control agencies use either simple interpolation methods or none at all in estimating fire danger between weather stations. We compare several methods of interpolation and use the fire weather index in the North Central Region of Ontario as a case study. Our work shows that the second order least square polynomial, the smoothed cubic spline, and the weighted interpolations had the lowest residual sum of squares in our verification scheme. These methods fit the observed data at both high and low fire weather index values. The highly variable nature of the spatial distribution of summer precipitation amount is the biggest problem in interpolating between stations. This factor leads to highly variable fire weather index fields that are the most difficult to interpolate. The use of radar and (or) satellite data could help resolve precipitation patterns with greater precision. These interpolation methods could easily be implemented by fire control agencies to gain a better understanding of fire danger in the region.

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Les organismes canadiens de protection contre les feux de forêt n'utilisent aucune méthode sinon des méthodes simples d'extrapolation pour évaluer les risques de feu entre les stations météorologiques. Nous avons comparé plusieurs méthodes d'extrapolation et utilisé l'indice forêt-météo dans le centre-nord de l'Ontario à titre d'exemple. Notre étude montre que les extrapolations au moyen d'un polynome du second degré utilisant les moindres carrés, d'une languette cubique flexible, et d'une pondération produisaient les sommes des carrés résiduelles les plus faibles selon notre procédure de vérification. Ces méthodes correspondent aux valeurs observées de l'indice forêt-météo autant élevé que faible. La très grande variation dans la distribution spatiale de la quantité de précipitation estivale est l'obstacle le plus important pour extrapoler entre les stations. Ce facteur entraîne une très grande variabilité dans les indices forêt-météo qui deviennent très difficiles à extrapoler. L'utilisation du radar et (ou) de données satellite permettrait de connaître la distribution des précipitations avec une plus grande précision. Ces méthodes d'extrapolation pourraient facilement être adoptées par les organismes de protection contre les feux de forêt pour obtenir une meilleure estimation des risques de feu dans la région.

[Traduit par la revue]

Introduction

Fire control agencies use a network of fire weather stations in their region to monitor the weather and to calculate the associated fire danger. In Canada, the Canadian forest fire weather index[1] (FWI) System (Canadian Forestry Service 1984; Van Wagner and Pickett 1985; Van Wagner 1987) is used to represent fire danger. The FWI is based on observations of temperature, relative humidity, wind speed at 10 m, and the 24-h precipitation at 12:00 local standard time (LST).

In Canada, fire control is the responsibility of each province and territory. Fire control organizations need to know the weather conditions and fire weather indexes between stations, but the density of fire weather stations is usually sparse. Currently, some agencies divide their province into regions and each region is then divided into a large number of cells. These cells are described using geographical coordinates systems such as latitude and longitude or Universal Transverse Mercator (UTM). Most cells are squares or rectangles that are 10–20 km on a side. Each cell is assigned to the most appropriate weather station and given the index values of that weather station. This cell assignment is a crude means of interpolating between stations and often leads to spurious sharp discontinuities in the index values as an artifact (Fig. 1). We felt that more sophisticated interpolation methods might yield more realistic fire weather index values. These more sophisticated methods blend information from many locations to estimate the FWI value between stations, as opposed to the cell assignment method, which uses only one location. To our knowledge a number of agencies and territories in Canada are not yet using an objective scheme to interpolate between stations. In this study, we investigated a number of interpolation methods to determine which gave the most accurate description of fire danger between stations.

Interpolation techniques (objective analysis) have been used extensively in meteorology since the 1950s. The methods used in this study are from a number of texts and papers (Essenwanger 1986; Goodin *et al.* 1979; Thiébaux and Pedder 1987). We chose a cross section of well-known interpolation methods used successfully in many applications. Our selections are by no means exhaustive and there are many other methods available (Lam 1983). To date, interpolation methods in the forest fire field have focused on the design of fire weather networks (Fujioka 1986; Furman 1975; King and Furman 1976) and the representativeness of weather stations (Lawson 1977; Turner and Lawson 1978). The impetus for our work was the fact that most operational

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[1] The FWI comprises three moisture codes and two intermediate indexes. The three moisture codes represent the moisture content of fine fuels (fine fuel moisture code, FFMC), loosely compacted decomposing organic matter (duff moisture code, DMC), and deep layer of compact organic matter (drought code, DC). The two intermediate indexes, which are derived from the moisture codes and the surface wind, indicate the rate of initial fire spread (initial spread index, ISI) and total available fuel (build up index, BUI). The two intermediate indexes are combined to obtain the FWI, which represents the intensity of the spreading fire.

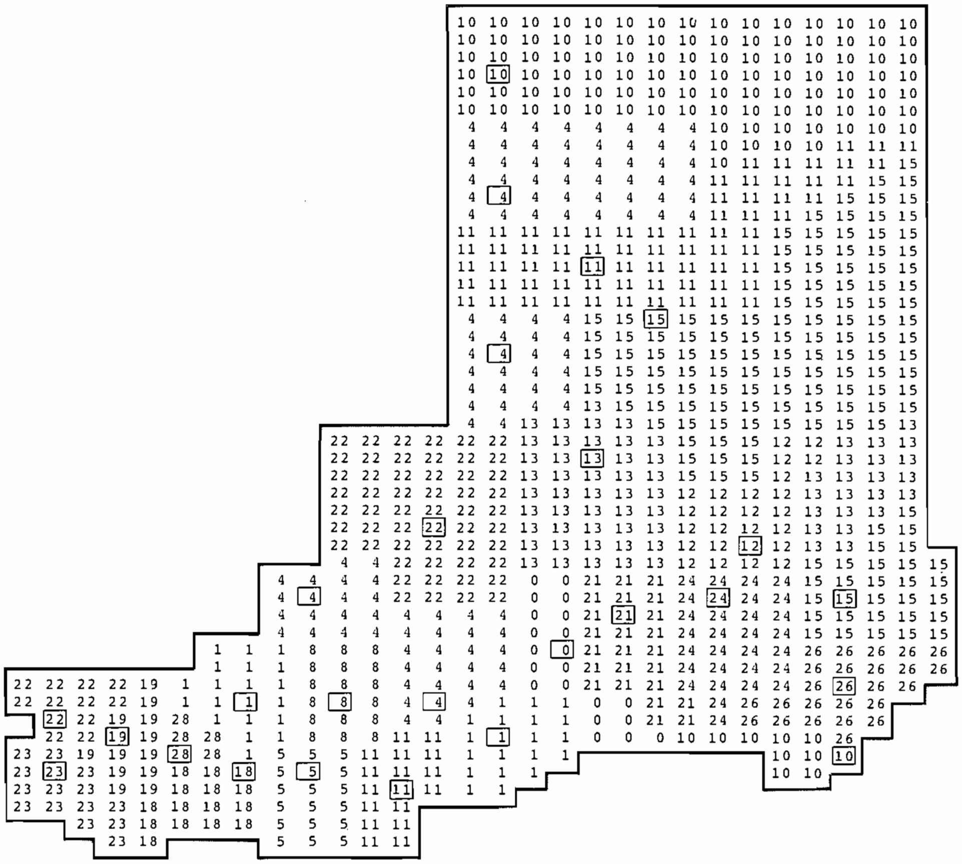

FIG. 1. The FWI field for June 23, 1987, using the cell replacement method. Cells with weather stations are boxed.

fire control agencies are not using any reliable interpolation schemes.

Data

We selected data from the North Central Region of Ontario (Fig. 2) for our study. This region, with an area of approximately 212 000 km^2, extends from the Ontario–Minnesota border and the north shore of Lake Superior to about 54°N latitude. The region had 26 weather stations in 1987 (Table 1 and Fig. 3). Each weather station recorded the 12:00 LST temperature, relative humidity, wind speed, and 24-h precipitation. Temperature and relative humidity were recorded on an Enercorp thermohygrometer. The 10-min averaged wind speed was measured at 10 m height by a 3-cup anemometer. Precipitation was measured with an Atmospheric Environment Service (AES) type B standard rain gauge.

The study extended over the period June 14 - August 18, 1987, when all 26 stations were operating. Although the interpolation methods studied could be implemented throughout the fire season, when not all of the stations were operating, only the days when all stations were operating were used to provide the best verification.

Method

We concentrated on the interpolation of FWI values. The accuracy of the methods was tested by removing one existing weather station (verification site) of the 26 stations and then using the remainder to estimate the FWI value at the verification site. The estimated value was then compared with the actual value.

FIG. 2. North Central Region, Ontario.

Eight weather stations were used as verification sites, one at a time, as a means of checking the goodness of fit of the interpolation methods. The sites selected were generally from interior locations because of a lack of data near the region's boundary. The residual sum of squares (RSS) was used to

TABLE 1. List of weather stations for 1987 in the North Central Region of Ontario

Station No.	Name	Location Lat. (°N)	Location Long. (°W)
1	Beaverhouse	48.539	92.063
2	Clearwater	48.961	92.013
3	Atikokan	48.758	91.588
4	GL Camp 515	48.630	91.080
5	GL Camp 517	48.412	90.394
6	Upsala-Camp 134	49.039	90.638
7	GL Camp 603	49.851	90.188
8	Shebandowan	48.617	90.188
9	GL Camp 234	49.090	89.960
10	Thunder Bay	48.398	89.337
11	GL Camp 45	49.242	89.163
12	Dorton	48.836	88.588
13	Armstrong	50.305	89.038
14	Macdiarmid	49.414	88.112
15	Ogoki Camp	50.150	86.580
16	Jellicoe	49.679	87.537
17	Miminiska Lake	51.601	88.563
18	Makokabatin Lake	51.773	87.262
19	Geraldton	49.789	86.887
20	Nakina	50.180	86.690
21	Klotz Lake	59.805	85.863
22	Manitouwadge	49.133	85.812
23	Pukaskwa	48.60	86.283
24	Summer Beaver	52.450	88.310
25	Landsdowne	52.226	87.863
26	Kasabonika	53.310	88.380

NOTE: GL, Great Lakes Paper Corporation.

rank performance. The RSS is equal to the absolute value of the difference between the estimated and actual values, squared.

$$[1] \quad \mathrm{RSS} = |\,\mathrm{ACTUAL} - \mathrm{ESTIMATED}\,|^2$$

The first interpolation method tried was the second-order polynomial fit. For this, two approaches were used to calculate the FWI. The first approach was to simply interpolate the FWI field, given 25 FWI values from the observing stations. The second approach was to interpolate temperature, relative humidity, wind speed, and 24-h precipitation and then to compute the FWI from existing equations.

The techniques used for estimating the FWI at the verification site are discussed in the following sections.

Second-order polynomial fit

A surface was fitted to the FWI (or weather variable) values for the 25 surrounding weather stations (i.e., 26 minus 1) using a least squares second-order polynomial. The technique requires minimization of χ^2, where

$$[2] \quad \chi^2 = \sum_{k=1}^{n} (\Delta Z)^2 = \sum_{k=1}^{n} (Z_k - a_1 - a_2 x_k - a_3 Y_k - a_4 x_k y_k - a_5 x_k^2 - a_6 y_k^2)^2$$

χ^2 must be a minimum where Z_k is the actual value at point (x_k, y_k) and n equals the number stations (25). The minimum value of χ^2 is found by setting the partial derivatives of χ^2, with respect to each of the coefficients a_i, equal to zero. For a second-order polynomial, six simultaneous equations must be solved. The value $Z(x,y)$ at any point can be calculated from

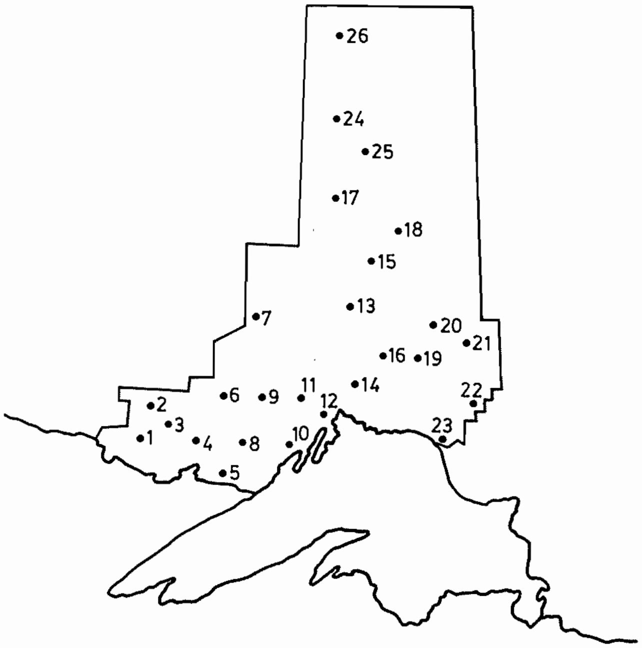

FIG. 3. The 26 weather stations are shown by number in a map of the North Central Region of Ontario. For station names and locations, refer to Table 1.

$$[3] \quad Z(x,y) = a_1 + a_2 x + a_3 y + a_4 x y + a_5 x^2 + a_6 y^2$$

For each day, a least squares second-order polynomial fit was performed, giving the values of $a_1 - a_6$ for eq. 3. The estimated FWI at the verification site was obtained by plugging the verification site location (x,y) into eq. 3. This same procedure was used to obtain interpolated values of temperature, relative humidity, wind speed, and precipitation. The interpolated weather variables were then input into a computer to derive the estimated FWI.

Cubic spline

Thiébaux and Pedder (1987) describe a two-dimensional cubic spline (surface), which is usually called a thin-plate spline (Duchon 1976).

The solution takes the form

$$[4] \quad Z(x_j,y_j) = \alpha_0 + \alpha_1 x_j + \alpha_2 y + \sum_{k-1}^{n} \beta_k \, \phi(r_{ij})$$

where

$$r_{ij} = [(x_i - x_j)^2 + (y_i - y_j)^2]^{1/2}$$

and

$$\phi(r_{ij}) = r_{ij}^2 \ln(r_{ij})$$

The coefficients α $(\alpha_0, \alpha_1, \alpha_2)$ and β $(\beta_1, \beta_2, \beta_n)$ can be found by solving the linear system

$$[5] \quad \overset{\mathbf{S}}{\begin{bmatrix} \Phi & A \\ A^T & 0 \end{bmatrix}} \overset{\mathbf{B}}{\begin{bmatrix} \beta \\ \alpha \end{bmatrix}} = \overset{\mathbf{C}}{\begin{bmatrix} Y \\ 0 \end{bmatrix}}$$

where Φ is a symmetric $n \times n$ matrix whose values are the

TABLE 2. Residual sum of squares for the various interpolation methods

	Second order		Cubic spline		Weighted							
						Gaussian						
Verification site (station No.)	FWI	Weather	Exact	Smoothed ($\theta = 5$)	Inverse square	0.1	0.01	Replacement[a]				
2	630	950	2038	721	802	906	942	3	1	6	23	—
								1716	946	2592	2766	—
3	1305	1273	2195	1314	1367	1290	1301	1	2	4	6	—
								1840	1716	3138	2150	—
4	2503	3603	4011	2553	2677	2449	2439	3	5	6	1	9
								3138	3588	5382	2876	2248
9	1411	1914	1669	1257	1350	1339	1349	6	8	11	7	10
								2654	1437	1532	2186	2368
11	575	437	596	533	571	610	614	12	9	14	10	—
								1417	1532	1969	1786	—
15	557	599	454	593	413	765	768	13	16	17	18	20
								3000	1236	2154	737	414
18	540	711	1190	440	662	704	709	25	17	15	20	—
								852	1537	737	711	—
19	1301	2373	1514	1304	1385	1217	1217	16	20	21	22	15
								1393	1891	1303	1397	2287

NOTE: —, there were no more nearby stations for the verification site.

[a] The number on the first line is the replacement station number; the number on the second line is the residual sum of squares.

$\phi(r_{ij})$ for all the combinations of observing stations (e.g., $i = 1$ to n and $j = 1$ to n) and $\mathbf{A}$ is an $n \times 3$ matrix for components of the first part of eq. 4 for all observing locations

$$\begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ \vdots & \vdots & \vdots \\ 1 & x_n & y_n \end{bmatrix}$$

and $\mathbf{Y}$ is an $n \times 1$ matrix of the observed values. The matrix $\mathbf{S}$ is an $(n+3) \times (n+3)$ matrix with every value on the main diagonal equal to 0.

Because $\mathbf{S}$ is a square matrix, a solution for β and α can be found that fits the observed values exactly. Thus, the thin-plate cubic-spline surface will yield the actual values at the observing locations, which is not the case for the polynomial fit. The solution of β and α is

$$[6] \quad \mathbf{B} = \mathbf{S}^{-1}\mathbf{C}$$

The constraint that the surface fit the observed data exactly can be relaxed. The thin-plate spline can be used to smooth the observed field by an approach similar to Reinsch (1967) for a one-dimensional spline. By smoothing we hoped to improve the overall fit of the estimated FWI field to actual surface, even though the result may violate the observed data. Smoothing is accomplished by adding a constant θ to the diagonal of the matrix Φ (e.g., $\Phi^1 = \Phi + \theta\mathbf{I}$, where $\mathbf{I}$ is an $n \times n$ identity matrix).

Weighted interpolation

The FWI value at the verification site in the region is estimated from a weighted average of surrounding stations within a radius of influence, R. Stephens and Stitt (1970) have shown the optimum radius of influence to be

$$[7] \quad R = 1.6\,(a/n)^{1/2}$$

where a is the area of the region and n the number of stations in the region.

The weighted average is calculated by

$$[8] \quad Z_{ik} = \frac{1}{\sum_{k=1}^{n} W_k(r)} \cdot \sum_{k=1}^{n} Z_k W_k(r)$$

where Z_k is the measured value at the kth measuring station, $W_k(r)$ the weighting function, and r the distance from the grid points to the station.

Three common weighting functions were tried:

(i) Inverse square: $W(r) = \dfrac{1}{r^2}$

(ii) Gaussian (0.1): $W(r) = e^{-0.1r^2}$

(iii) Gaussian (0.01): $W(r) = e^{-0.01r^2}$

Wherever $r > R$, $W(r) = 0.0$.

Replacement

Currently in Ontario, cells within a region are assigned the FWI value from a nearby weather station. The nearest station is not used necessarily, but the most appropriate based on site, topography, proximity to water, and experience. The assignment of cells to a weather station is done subjectively based on the experience of regional fire control personnel. We tried in our study to use all reasonably close stations (4 or 5) as an estimate for the verification site; this included the assignment made by regional fire control personnel.

Results

The results are summarized in Table 2. The RSS for the two approaches for the second-order polynomial fit, namely the interpolated FWI and the interpolated weather to calculate the FWI, are listed in the first two columns of Table 2. The first approach, interpolating FWI, on average best represented actual values (lower RSS). We investigated the cause for the higher RSS of the interpolated weather inputs to estimate the FWI. The interpolated fields for temperature, relative humidity, and wind speed fit

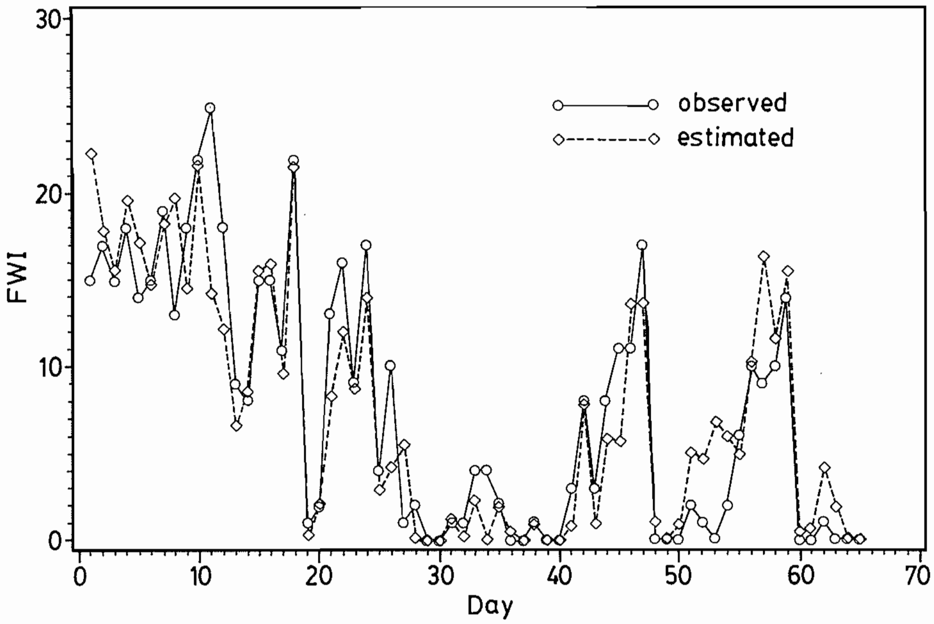

FIG. 4. Plot of actual FWI and estimated FWI versus day (June 14 – August 18, 1987) for the least squares second-order polynomial method at verification site 9.

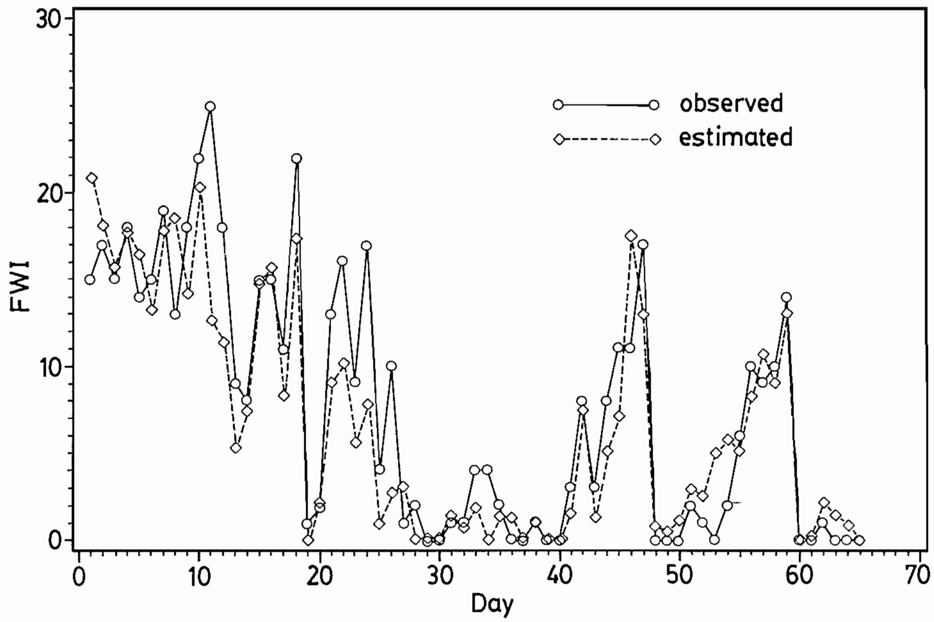

FIG. 5. Plot of the actual FWI and estimated FWI versus day for the smoothed cubic spline at verification site 11.

reasonably well (low RSS); however, the 24-h precipitation had a high RSS relative to other weather variables.

The first column under the cubic-spline heading in Table 2 shows the surface fitting the observed data exactly, whereas the second column allows the surface to smooth observed values. The smoothed cubic spline is the better fit. The value of $\theta = 5$ for the smoothed spline was arrived at through trial and error. Values of θ from 0 to 75 were tried. Selecting the best θ was a compromise; we kept the RSS as low as possible at the verification sites for the eight cases and maintained the smoothed surface values reasonably close to the observed FWI values at the other 25 stations. When $\theta = 5$, the root mean square error between the 25 actual FWI values and the estimated spline values for the eight test cases was 2.4. For comparison, the second-order polynomial had a root mean square error of 2.8 for the same eight cases.

The three columns under the weighted heading represent three different weightings, namely inverse square, Gaussian 0.1, and Gaussian 0.01. The radius of influence used in the weighted interpolations was 145 km and was calculated using eq. 7. We tried values larger and smaller than 1.6 (1.0–2.0) in eq. 7, which changes the radius of influence. However, the RSS using the different radii were similar. This implies that for our network the weighting method is insensitive to changes of the radius of influence in the 90- to 180-km range.

The last five columns of Table 2 show the RSS for the replacement station number as well as the RSS listed for each verification site.

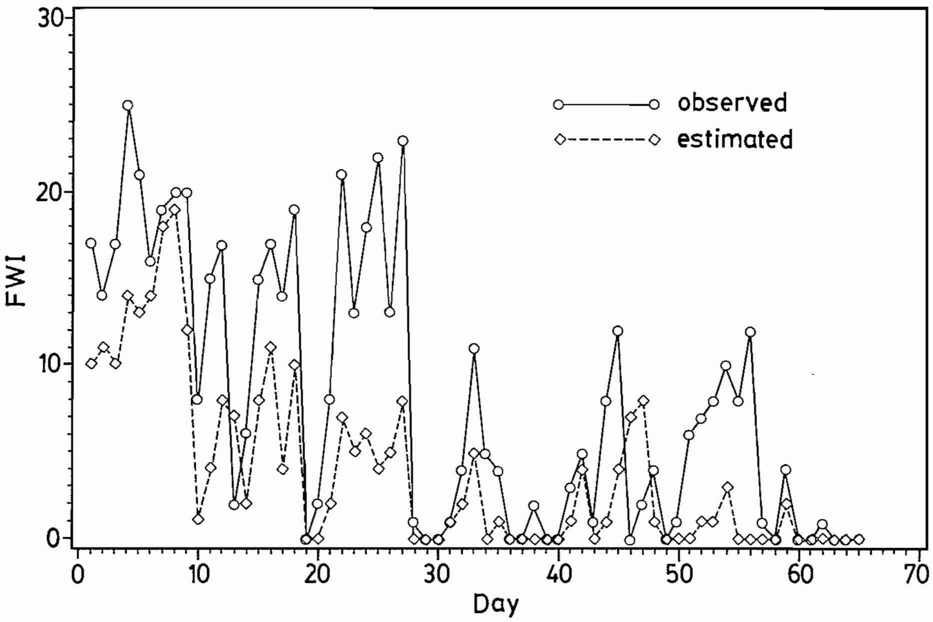

FIG. 6. Plot of the actual FWI and estimated FWI versus day for the replacement method at verification site 9 (estimated with data from station 6).

The RSS is used as a relative measure to rank the performance of each interpolation method. There was concern that there might be a high nonconstant correlation between data points because we used consecutive days. This may have affected the ranking of the methods. As a result, the estimated and actual values of FWI were plotted for each method. Figure 4 shows a plot that is representative of the majority of the graphs. The estimates in Fig. 4 appear to be reasonable approximations of the actual data, although there is a tendency to underestimate the high FWI values. Figures 5 and 6 are typical examples of a low and high RSS, respectively. A study of the figures indicates that the interpolation methods seem to fairly accurately estimate FWI values at both the high and low ends.

Discussion

Table 2 shows that the smoothed cubic spline, the second-order polynomial, and the weighted interpolations were consistently the best methods for interpolation. Of the three, the smoothed cubic spline was the most consistent and the replacement method had the highest RSS compared with the others.

The high RSS associated with the replacement method were expected. The replacement method is a crude means of interpolating data and fared poorly relative to the other interpolation methods. There were only a few verification sites where one or more of the replacement stations compared well with the other interpolation methods, but these were the exceptions.

The thin-plate cubic spline with an exact fit to the observed data resulted in high RSS for some of the stations. The poor fit is a result of trying to fit a FWI field with large variations, resulting in a fitted surface with large amplitude and associated large errors.

Considerable variation in FWI over short distances was often the result of a highly variable precipitation field. Rain showers are typical of summer precipitation. Showers can result in some weather stations receiving a significant rainfall (≥ 15 mm) whereas nearby stations receive no precipitation. The effect of significant precipitation has a large impact on the FWI. Figure 7 is an example of a highly variable FWI with the corresponding precipitation (Fig. 8) being largely responsible for variation in the FWI. The highly variable precipitation field also caused the poor performance of the interpolating weather method in estimating the FWI. This suggests that a better knowledge of the precipitation field is critical to understanding and predicting the fire weather and associated indexes. Hence, there have been investigations by a number of fire control agencies concerning the use of weather radar in monitoring areal extent and amount of precipitation. Up until the writing of this report, no agency has used radar precipitation in calculating the FWI. The number of weather stations in the North Central Region is adequate for the interpolation of temperature, relative humidity, and wind speed. However, the poor performance of the interpolating weather method in estimating the FWI shows that the density of weather stations is inadequate for precipitation monitoring. The inability of the monitoring network to ascertain the spatial variability of precipitation impacted negatively on the performance of all interpolation techniques.

The nature of the precipitation, whether spotty or widespread, can also be used by fire control agencies to assign a confidence rating for the interpolated data. If there is no precipitation, confidence in the interpolation is high. When precipitation is widespread, the confidence is also high for interpolating between stations. Conversely, when precipitation is spotty, confidence in interpolation is low. Widespread precipitation is associated with well-defined synoptic systems (e.g., lows and troughs), whereas spotty precipitation is associated with isolated showers and thunderstorms. Radar, or even satellite data, could be used to determine the nature and occurrence of precipitation.

The weighted interpolations predicted well compared with other interpolation methods. As mentioned earlier, the radius of influence was approximately 145 km. Turner and

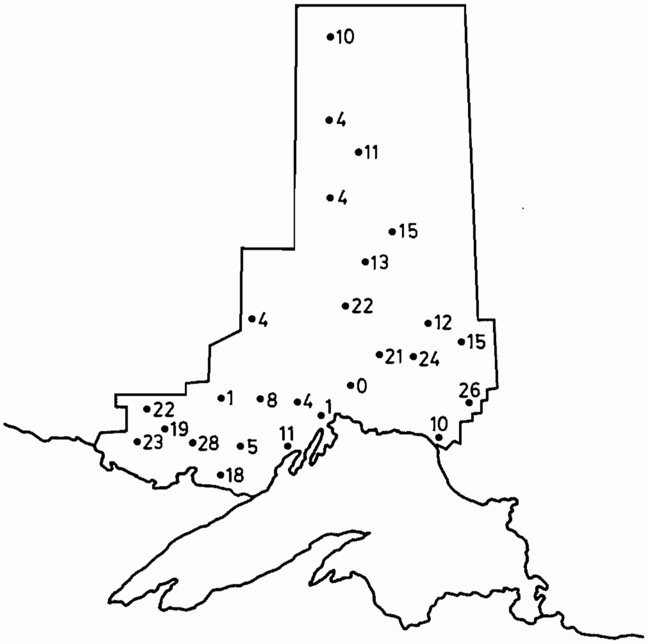

FIG. 7. The observed FWI for June 23, 1987.

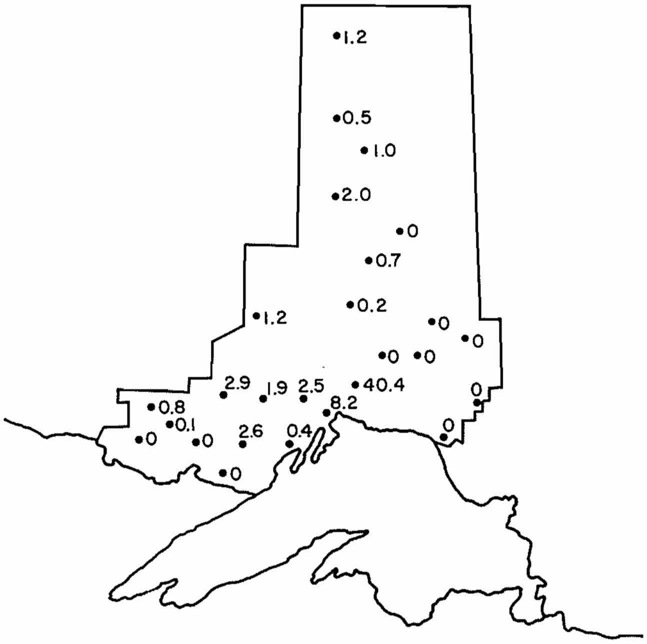

FIG. 8. The observed 24-h precipitation for 1200 LST June 13, 1987.

Lawson (1978) state that the FWI value is highly reliable within 40 km of the observing station and unreliable beyond 160 km. This is due to the spatial variability of summer precipitation. Given the paucity of stations in the network, using a 40 km radius of influence is not workable. This region would require 340 weather stations to have a radius of influence of 40 km. The distribution of weather stations in the North Central Region of Ontario is uneven, with few stations in the northern sections. This is fairly typical of forest fire control regions in Canada. The lack of weather stations in the north impacts negatively on the success of interpolation techniques.

The weighted interpolating methods have the added benefit, compared with the second-order polynomial and smoothed cubic-spline methods, of the final field fitting observed data. The drawback is that for each cell the weights have to be calculated individually, as opposed to the polynomial and spline methods where only the position has to be substituted into an equation to estimate FWI value.

Missing weather observations will not hinder the use of the polynomial cubic spline or weighted methods, though we would expect poorer results with fewer weather stations. Missing observations would require new cell assignments for the cell replacement method. Using spatial interpolation methods is one possible way of estimating the missing observation values.

Prior to using either the polynomial or smoothed cubic-spline methods, it is necessary to discuss the subject of smoothing observations. The root mean square error of the estimated FWI for both methods is in the 2.4 to 2.8 range. Also, smoothing the observed FWI tends to underestimate the extreme high values and overestimate the low values. The sensitivity of the FWI is such that a change of 3 in the FWI value is hardly distinguishable. Also, as the results demonstrate, the smoothed cubic spline is superior to the exact-fit cubic spline. If operational personnel are reluctant to smooth actual observations, the graphical output of the FWI could include the smoothed field with an overlay of actual values at the observing stations. Any large variation could be flagged by the computer and alternative procedures made available (e.g., weighted interpolation or exact cubic spline). We feel that any of the interpolation methods could be implemented with relative ease into field operations. Most fire control agencies are computerized; therefore, implementation could be achieved by simply adding a program module for interpolation. It would be prudent to have additional weather observations from surrounding regions when implementing an interpolation method to eliminate boundary problems. Some fire control organizations are experimenting with geographic information systems to tie in with interpolation methods.

Fire control agencies are mostly concerned with days of high fire danger. From Figs. 4 and 5 we see that when the FWI is high (13 or greater) better interpolation methods reasonably mimic the actual FWI. Fire control agencies, however, also want to have confidence in the method when interpolation suggests low FWI values. They do not want false alarms (high FWI estimate when actual FWI is low).

There are other, more sophisticated, interpolation techniques than were used in this study. Statistical linear interpolations, known as "kriging" (Matheron 1981) and optimal statistical objective analysis (OSOA) (Thiébaux and Pedder 1987), are two such methods. They should be superior to the smoothed spline surface when reliable covariance estimates are available. However, in our study, we felt that a prescribed model for spatial covariance, as required by the Kriging and OSOA methods, would not be reliable because the weather station network in the North Central Region is varied in instrumentation and location. These methods would be worth investigating at some later date when the fire weather network (location and equipment) is stable.

Table 2 shows that test sites 3, 4, 9, and 19 had higher RSS than the remaining four test sites. The RSS of station 4 was significantly higher than that of the others. This suggests that there is either a marked difference in weather at this location compared with surrounding areas or that there is a physical problem with the station. Physical problems may include poor site, poor equipment, and human errors. We feel that there is probably a physical problem with site 4, as there should be no marked difference in weather at site 4 compared with surrounding sites. All verification sites with high RSS should be checked to ensure that no physical problems are responsible for poor results.

Readers should note that our study focused on the FWI. However, preliminary work done with moisture codes and subindexes of the FWI show results similar to those obtained in this study. We would suggest that interpolating FFMC, ISI, or any other component of the FWI would yield results similar to those obtained by us for the FWI.

Summary

The present study has shown that, in the North Central Region of Ontario, the use of interpolation methods to estimate FWI values between observing stations would more realistically represent fire weather. Our tests, accomplished by removing observation stations (verification sites) from the data set and then comparing the actual versus estimated results, revealed that three methods, the least squares second-order polynomial, smoothed thin-plate cubic spline, and the weighted interpolations, fit the data best. Of the three, the smoothed cubic spline was the most consistent. These methods eliminate the problem of the unnatural sharp differences in the FWI field generated by the replacement method.

Finally, the study suggests that the spatial variability of summer precipitation is the biggest unknown in interpolating FWI between weather stations. The current density of weather stations (rain gauges) does not allow a detailed knowledge of the precipitation field. Remote sensing via satellite or radar data may provide the necessary detail.

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